$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \dots$$

25.

$f(x) = e^{\frac{x}{2}}$ $\quad f(0) = 1$ $\quad f'''(x) = \frac{1}{8}e^{x/2}$

$f'(x) = \frac{1}{2}e^{x/2}$ $\quad f'(0) = \frac{1}{2}$ $\quad f^4(x) = \frac{1}{16}e^{x/2}$

$f''(x) = \frac{1}{4}e^{x/2}$ $\quad f''(0) = \frac{1}{4}$

$$f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3$$

general term

$$1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{8 \cdot 3!} + \frac{x^4}{16 \cdot 4!} + \dots + \frac{x^n}{2^n \cdot n!}$$

Feb 12-9:59 AM

general term

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \frac{x^n}{n!} + \dots$$

$$e^{x/2} = 1 + \frac{x}{2} + \frac{\left(\frac{x}{2}\right)^2}{2} + \frac{\left(\frac{x}{2}\right)^3}{3!} \dots$$

$$= 1 + \frac{x}{2} + \frac{x^2}{2^2 \cdot 2} + \frac{x^3}{2^3 \cdot 3!}$$

Feb 12-10:09 AM

b) $\dfrac{e^x-1}{x} = \sum_{n=1}^{\infty} \dfrac{x^{n-1}}{n!}$ or $\sum_{n=0}^{\infty} \dfrac{x^n}{(n+1)!}$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$

$e^x - 1 = x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$

b) $g(x) = \dfrac{e^x-1}{x} = 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} \cdots + \frac{x^{n-1}}{n!}$ n starts at 1

$\frac{x^n}{(n+1)!}$ n starts at 0

c) $g'(x) = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} + \frac{4x^3}{5!} \cdots$

$x=1$ $\quad g'(1) = \frac{1}{2} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} \cdots \overset{?}{=} \underline{1}$

$= \sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$

$g'(x) = \dfrac{xe^x - (e^x-1)}{x^2}$ $\quad g'(1) = \dfrac{e-(e-1)}{1} = \underline{1}$

Feb 12-10:13 AM

24. $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \cdots + \frac{x^n}{(n+1)!}$

$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$f'(0) = \frac{1}{2}$

$\frac{f''(0)}{\cancel{2}} = \frac{1}{3!} \cdot 2 = \frac{1}{3}$ $\qquad \frac{f'''(0)}{\cancel{3!}} = \frac{1}{4!} \cdot 3! = \frac{1}{4}$

$f^{(10)}(0) = \frac{1}{11}$

Feb 12-10:17 AM

9.2 b Taylor series

Maclaurin Series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^3(0)}{3!}x^3 \dots$$

works for x near 0

for x near "a"

$$\boxed{f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 \dots = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n}$$

Feb 12-10:32 AM

find the Taylor Series for $f(x) = e^x$ about $x=2$

at $x=2$

centered on $x=2$

$a=2$

$f(x) = e^x \quad f(2) = e^2$

$f'(x) = e^x \quad f'(2) = e^2$

$f''(x) = e^x \quad f''(2) = e^2$

$\vdots \quad \vdots \quad \vdots$

$$e^x = e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 + \frac{e^2}{3!}(x-2)^3 + \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{e^2(x-2)^n}{n!}$$

Feb 12-10:39 AM

1st 3 nonzero terms

find the Taylor Series for $f(x)=\cos x$ at $x=\frac{\pi}{4}$

$f(x)=\cos x \quad f\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$f'(x)=-\sin x \quad f'\left(\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$

$f''(x)=-\cos x \quad f''\left(\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$

$$\cos x \approx \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\frac{\sqrt{2}}{2}}{2}\left(x-\frac{\pi}{4}\right)^2$$

Feb 12-10:46 AM

$$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{4!}-\ldots$$

$1+(2+3)$

$\frac{2+3}{4}$

find the mac series for $\frac{1+\cos(2x)}{2}$

$$\cos(2x)=1-\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}\ldots$$

$$\frac{1+\cos(2x)}{2}=\frac{2}{2}-\frac{(2x)^2}{2\cdot 2}+\frac{(2x)^4}{2\cdot 4!}$$

Feb 12-10:52 AM

find the mac series for xe^x

$$xe^x = x\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots\right)$$

$$xe^x = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \ldots + \frac{x^{n+1}}{n!}$$

Feb 12-11:00 AM